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SCHOOL SUBMODEL FOR LARGE URBAN SCHOOLS.

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A SUBMODEL OF THE MODEL DEVELOPED IN TECHNICAL NOTE 24, "URBAN EDUCATION SYSTEMS ANALYSIS," (A TOTAL DECISIONMAKING PROCEDURE FOR THE ALLOCATION OF RESOURCES FOR LARGE EDUCATIONAL FACILITIES) IS FURTHER SPECIFIED. THE SCHOOL SUBMODEL IS CONCERNED WITH THE DEFINITION OF THE BASIC INPUT DATA REPRESENTING EDUCATIONAL POLICY ON FACILITIES, STAFF, AND PROGRAMS. THE OBJECTIVE OF THIS PAPER IS THE SPECIFICATION OF THESE INPUTS, THEIR INTERRELATIONSHIPS, AND THE PRESENTATION OF THE DATA IN THE FORM NECESSARY FOR THE LATER EVALUATION OF COSTS AND EFFECTIVENESS. FOUR TYPES OF INFORMATION ARE GENERATED FROM THE MODEL FOR USE IN EDUCATIONAL POLICY--(1) FACILITY REQUIREMENTS IN TERMS OF TOTAL SCHOOL PLANT SIZE AND FUNCTIONAL SPACE ALLOCATION, (2) STAFFING REQUIREMENTS BY NUMBER AND OCCUPATIONAL CATEGORIES, (3) SPECIAL PROGRAM REQUIREMENTS IN TERMS OF STAFF AND SPACE, AND (4) STAFF AND SPACE IMPLICATIONS OF SCHEDULING MODIFICATIONS. (HW)

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SCHOOL SUBMODEL FOR LARGE URBAN SCHOOLS

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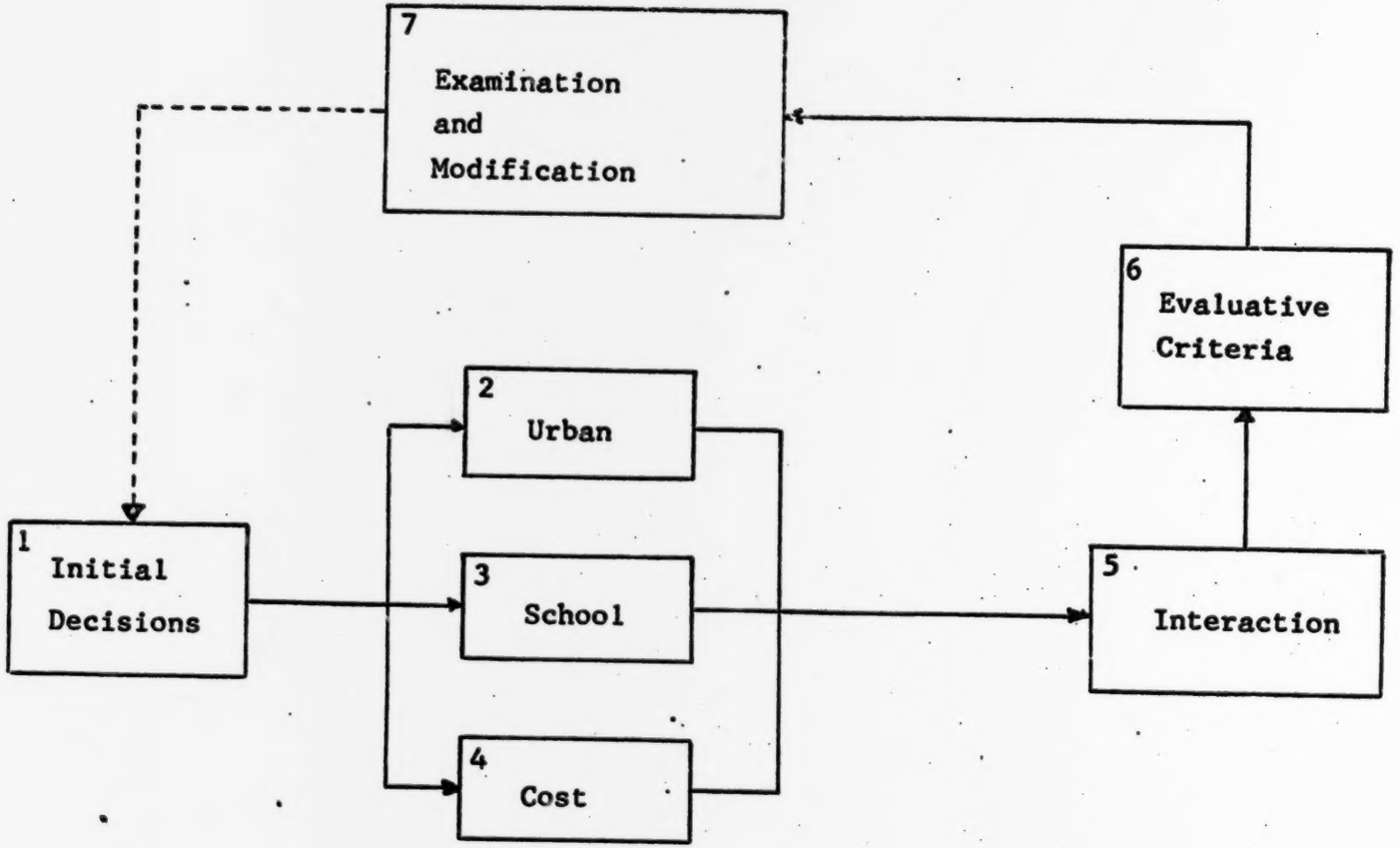
In Technical Note 24, Urban Education Systems Analysis, a framework was described of a decision-making procedure for the allocation of resources for large educational facilities. An overview of that decision-making procedure is shown in Figure 1 and will be briefly restated. An initial educational policy relative to school size, location and facilities is proposed. This policy is evaluated relative to the urban environment (Box 2) as measured by the location and demographic characteristics of the student population and the school environment (Box 3) as measured by the facilities, staff and programs of the school plant. The cost (Box 4) incurred by the construction and annual operation of the proposed educational system is then estimated. The interaction (Box 5) of the above elements as measured by operational indices of effectiveness and cost are evaluated relative to educational goals and objectives (Box 6). Since these goals and objectives are multi-valued, it is anticipated that no one policy is likely to be optimal relative to all objectives, and therefore several modifications (Box 7) on the initial policy will be made until a final policy is selected.

The cost submodel of that total decision-making procedure was further described and specified in Technical Note 30, Cost Model for Large Urban Schools. This present note is a further specification of the school submodel of the total decision-making procedure. The school submodel is concerned with the definition of the basic input data representing educational policy on facilities, staff and programs. The specification of these inputs, their interrelationships and the presentation of the data in the form necessary for the later evaluation of costs and effectiveness is the objective of this note. Of concern also is the selection of a level of aggregation of model definition that is feasible from a data-gathering viewpoint and which affords the necessary sensitivity for selection among educational policies. This necessarily depends on the existing data base and judgment.

It is intended also to introduce some techniques that may be of use in the specification of input data and in defining their interrelationships. The approach presented of a general formulation of the input data, in itself, allows, as do all models, for the assessment of changes in basic educational policy which the input data represents. It is through this investigation of the effect various levels of the inputs have on the process, in this case the educational process, that one learns more about the system and in turn learns more about the determination of effective educational policy.



URBAN EDUCATION SYSTEMS ANALYSIS

Basic Model

Figure 1

Facilities

This portion of the school submodel is concerned with the specification of the functional areas within the school plant. This specification consists of the identification of the facility or service provided and the determination of space, in floor area of the school plant, required to provide that service to the student population. The basic inputs are the type of facility and the space allocated to that type on a per pupil basis. The output is the total floor area of building space required.

The approach used herein for the determination of the required space is defined by the following categories:

1. Regular instructional area
2. Supplemental instructional area
3. Service and structure area.

One may approach this problem, that is the estimation of space needs, on strictly a statistical basis. The method would be to take a statistical sample of existing school plants, measure space allotted to the different functional categories, measure school enrollment size and relate these measures by the development of an empirical functional form. This is a reasonable approach and sources of data are available to develop some of these relationships. In the use of such data one assumes that the curriculum and scheduling of the curriculum elements implicit in the data are applicable to the proposed facility. Further it assumes one may extrapolate that data (since it is also desired to estimate space requirements for facilities larger than those currently in existence) beyond the range of the plant sizes sampled.

The first of these assumptions is not expected to appreciably affect the precision of the space requirements determined. Some care should be exercised, however, in the extrapolation of data to school plants appreciably beyond the size of the school plants sampled. Usually some knowledge of the process independent of the sample data is necessary to safeguard the validity of the extrapolation.

The approach presented in this note will be partially statistical and partially judgmental. This is undertaken both due to the limitation of the strictly statistical approach mentioned above and due to the requirement to demonstrate the dependency of space requirements on educational input parameters. Educational policy that affects these input parameters may then be assessed. In the determination of the space requirements for the regular instructional area the method employed will essentially be statistical in nature, and at level of aggregation of space per pupil

factors. The supplemental area requirements will be determined in terms of the input parameters of classroom and curriculum scheduling requirements, assuming these are known and specified as input by the educational decision-maker. The service and structure area requirements will be determined through the development of strictly statistical factors. These approaches will now be discussed.

The regular instructional area which is synonomous with classroom area is assumed to be that part of the school plant which, if required, could provide space for the entire student population; that is, each student would have a desk and chair area. It is also space that can be effectively used for instructional purposes independent of the particular course of instruction. The space requirements generated by this category therefore depend on the total number of students and the preassigned space per pupil. This input value of space per pupil may be based on educational policy, statistical data, judgment or human engineering study. It is a function of school level, and provision will be made to consider three school levels, (1) elementary, defined as grades, one through four, (2) intermediate, grades 5 through 8, and (3) secondary, grades 9 through 12.

The floor area required to service the regular instructional program is therefore

$$A_c = \sum_{i=1} N_i \, g_i$$

where

A_c = total regular instructional floor area

N_i = number of students in the i^{th} school level

g_i = area per pupil of regular instructional space for the i^{th} school level

i = index of school levels.

Supplemental instructional areas are administrative and adjunct areas such as laboratories, auditoriums, gymnasiums and cafeterias that are used only a portion of the time and usually by only a portion of the student body. Supplemental instructional areas considered at this point are those in which the number of students (or school staff) to use the facility are known and the frequency of scheduling of the use of the facility and the availability of the use of the facility during the school day are specified as input data. The determination of requirements when the facility is in support of an elective program, i.e., where the potential users are specified in only a probabilistic way, will be considered later.

The basic approach in determining supplemental instructional area requirements will be the same whether discussing a facility such as a laboratory, cafeteria or auditorium. It is assumed that the space requirement per facility is given by the following function:

$$A_j = (f_j/p_j)\ N_j q_j$$

where

A_j = floor area required for the j^{th} facility

f_j = frequency of scheduling of the use of the facility in periods per day based on curriculum requirements

p_j = available periods per day for the scheduling of the use of the facility

N_j = number of students that use the facility

q_j = space required in floor area per pupil.

The explanation of some of the parameters of the above formula is as follows. First the value q, which is specified as input, is the floor space required per pupil in the effective use of the facility. Guidelines on the value of this parameter may be obtained through statistical estimation, though it is thought of here as based more on educational and engineering judgment. It is the space per pupil such that in utilizing the facility the program of instruction may be effectively carried out. The value f is determined, where applicable, by course requirements, e.g., physics lab is taken twice per week, gym once per week, or by the nature of the facility such as the cafeteria is used five times per week. The value p is constrained by the length of the school day, and the length of the block of instruction, e.g., a gym period is a 2-hour block and may be given anytime during a 6-hour school day and therefore the available periods of use per day is 3. The value p is also constrained by the nature of the facility, e.g., the cafeteria may only be thought of as available over a small range of periods covering the noon hour.

The rationale of the formula is as follows. If N pupils are eligible to use the facility then the required facility size would be of maximum size Nq. This area Nq would be required if the facility were to be utilized at one time by the eligible students N. This maximum size can obviously be reduced by scheduling the use of facility p periods per day. The required size would then be (1/p)(Nq). The requirement that each student use the facility f times per day would inflate the space requirements to (f/p)Nq or the above formula.

A plot of the function f/p is shown in Figure 2. This chart indicates the significant effect scheduling and periods of use have on the total space requirements.

The above supplemental instruction areas are (if q is selected reasonably) to be considered minimum requirements, that is, if one were able to optimally utilize the particular facility the above formula would yield the required floor space. It may be desired to constrain the range of these requirements based on other considerations such as multiple use of the facility. For example, if the gymnasium is used in support of school games as well as in support of the regular curriculum, a minimum size may be postulated based on the larger of these values. Similarly the effect of the above formula is, in the use of facilities in support of the curriculum, to set class section size and this may be constrained at a minimum size for the effective use of teaching personnel.

The values N, q, f and p will be specified relative to the use of a particular facility and the total supplemental area requirements, A_e, will be determined by summing over the n defined supplemental areas or

$$A_e = \sum_{j=1}^{n} A_j = \sum_{j=1}^{n} (f_j/p_j)\ N_j\ q_j$$

The final category considered in the determination of space requirements is service and structure area. This will be estimated by assuming that the ratio of service and structure area to total floor area is approximately constant. This ratio will be statistically determined from sample data.

Therefore

$$A_S = rT_A$$

where

A_S = service and structure area

r = constant ratio to be estimated

T_A = total floor area requirements

and the total floor area of the plant is estimated from

$$T_A = \frac{A_c + A_e}{(1 - r)}$$

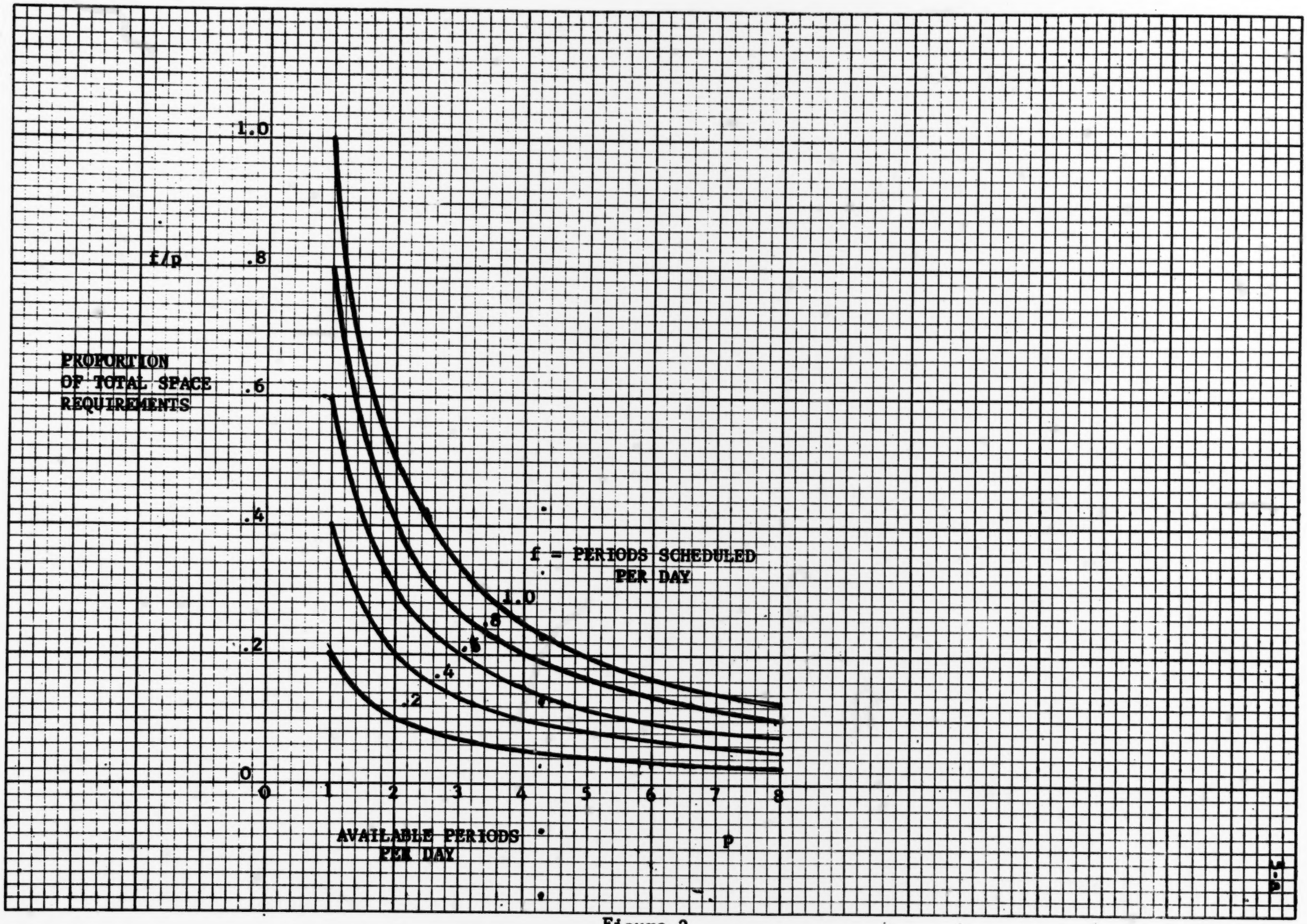

1.0
f/p
.8
PROPORTION
OF TOTAL SPACE
REQUIREMENTS
.6
.4
.2
0
0
1
2
3
4
5
6
7
8
f = PERIODS SCHEDULED
PER DAY
1.0
.8
.6
.4
.2
AVAILABLE PERIODS
PER DAY
p

Figure 2

This output of the school submodel will be input in the cost model for the determination of costs. The facilities themselves, that is, their existence or nonexistence, will be input into the effectiveness determination of the school facility.

Staffing Requirements

The staffing requirements will be based on staffing ratios applied to specified occupational categories such as:

1. Administrators
2. Teachers
3. Teaching aides
4. Administrative Secretaries and Clerks
5. Instructional Secretaries and Clerks
6. Health Personnel
7. Operations Personnel
8. Maintenance Personnel

A functional form is hypothesized that will allow for diseconomies and economies of scale in staff requirements as a function of student population size. The particular functional form is:

$$T_s = \sum_{u=1}^{3} \sum_{v=1}^{8} e_{v,u} N_u^{f_{v,u}}$$

where

T_s = total staff required

$e_{v,u}$ = parameter of staffing function for the v^{th} staff category and the u^{th} school level

N_u = number of students at the u^{th} school level

$f_{v,u}$ = parameter of staffing function for the v^{th} staff category and the u^{th} school level

The value of the exponent f in the above formula indicates the economies or diseconomies of scale in staffing requirements. For an f value of one, the formula implies a proportional relationship between staff and enrollment, that is a constant staff student ratio (measured by the parameter e). A value of the parameter f less than one (and greater than zero) indicates a staffing requirement that is increasing at a decreasing rate with respect to enrollment. An f value greater than one indicates a staffing requirement that is also increasing with enrollment but at an increasing rate. This would reflect the case where increased organizational complexity (with increased enrollment) generates additional manpower requirements. These relationships are allowed to vary with respect to school level and occupational category.

The parameters of the above equation-$e_{v,u}$ and $f_{v,u}$-may be estimated through regression analysis of sample data relative to a given region or may be specified as inputs reflecting a specific educational policy. Investigations may thus be made based on the manpower resource requirements that are generated by educational policy affecting staffing patterns such as student-teacher ratios.

Programs

In the above specification of staff the relationship to specific programs was not explicitly stated. These relationships are implicit in the staffing ratios. It may be of interest to estimate resource requirements specifically for a given curriculum element or program, such programs that may require a specialist and are elective in nature so as to be selected by only a small proportion of the student body. Programs such as guidance, language, and honors programs, may be in this category. A technique is presented that may be of use in estimating the number of students likely to participate in such programs and therefore the staff and the facility space requirements induced by these special programs. These requirements will be added to the staff and space requirements developed previously.

This will be looked at from two viewpoints: (1) what student body is required to support a program efficiently, and (2) what utilization will be made of a program by a given size student body? The first viewpoint is of interest in planning the size of a school facility and the second in evaluating the utilization of a program by a school of a given size.

The statement of the problem is as follows:

Let

p = probability that an individual student will take a given course. This may be based on historical data of elective course selection such that

$$p = \frac{E_1}{E} .$$

where

E_1 = total number of students taking a given course

E = total number of students eligible to take a given course

Then the probability P that a class of size M or larger will result from a student population of size N, is approximately

$$P = \sum_{x=M}^{N} \binom{N}{x} p^x (1-p)^{N-x}$$

One problem posed then is to find the student population, N, given the required class (or program) size, M, and further, given that one wants to run only a small risk of not having at least the required class size elect to participate in the program. If extensive binomial probability tables were available, the table would be searched for the required N. However, for the size of p (small) and N (large) contemplated a reasonable approximation to the above probability function is

$$P = \sum_{x=M}^{N} \frac{(Np)^x e^{-Np}}{x!}$$

This function, the Poisson probability distribution, is suggested because it can more easily be evaluated and graphically displayed. For example, the relationship between class size and student population is shown in Figure 3. This curve has been constructed so that there is a 2%(P=95,5) chance that out of the student population of size N a class of size M or larger will choose the given course of instruction. Similar curves may be drawn for other probability levels, e.g., 90%, 1%.

The use of this curve will be demonstrated by the following example. Let us assume that to support, full time, one French teacher, three classes per day of class work is required. If French is given three times a week then 5 individual groups of students are required and if class size is 20, then 100 students are required to support the program, i.e., one French teacher. The problem then is, with 95% assurance, what size student body will be required to have 100 or more students selecting French. Based on past experience only 1% of the students have selected French. In summary then

P = .95

p = .01

M = 100

N = ?

PROBABILITY RELATIONSHIPS
BETWEEN PROGRAM SIZE, M
AND STUDENT POPULATION SIZE, N
(CUMULATIVE POISSON DISTRIBUTION)

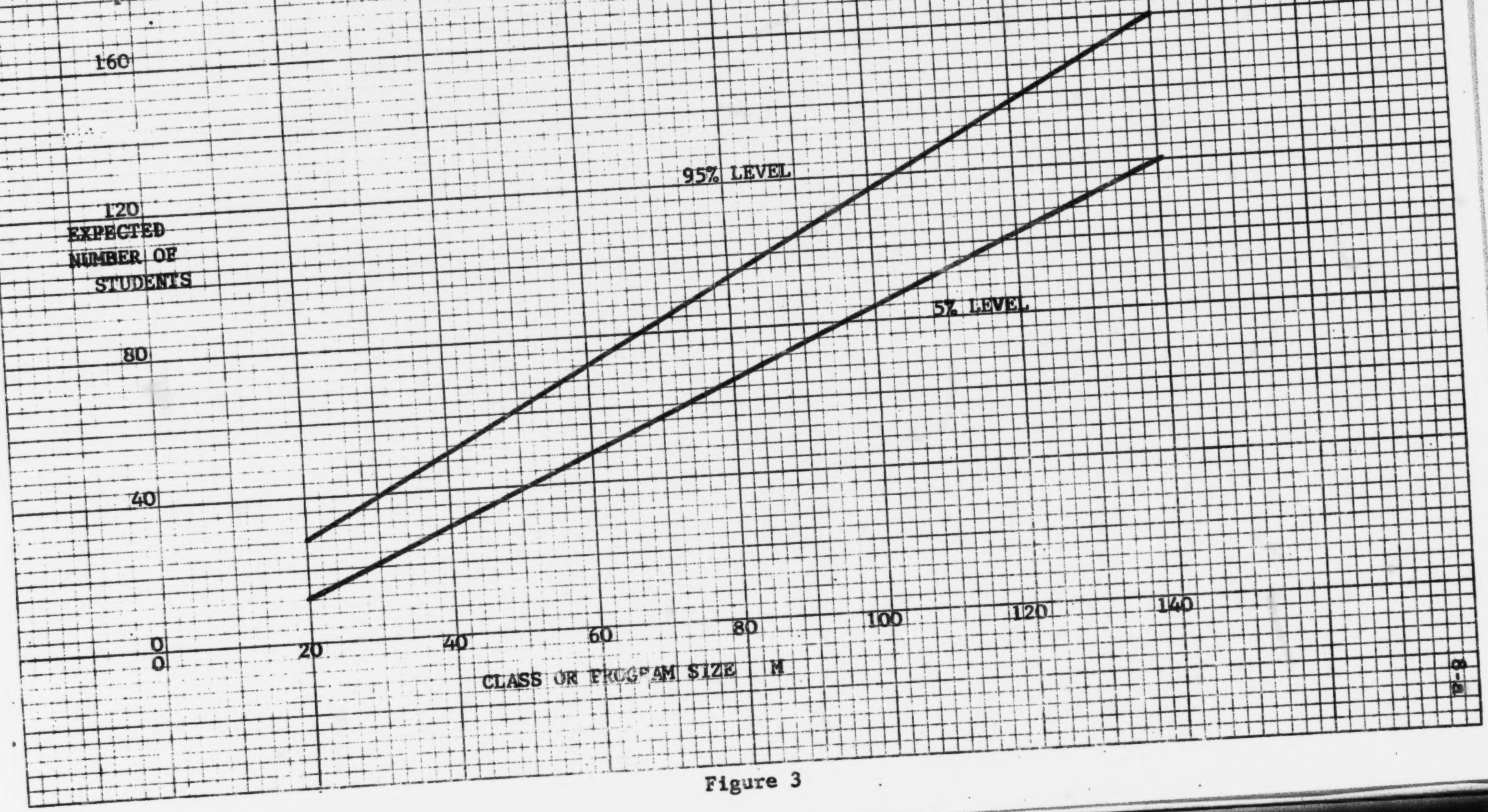


Figure 3

Entering the abscissa of the curve marked 95% shown in Figure 3 with M = 100, the value Np = 116 is read on the ordinate. For the value p = .01 then N = 116/.01 = 11,600. A student population of 11,600 is required to have a high assurance (95%) of the capability of supporting a French teacher full time. One is able thus to develop some measure of the student population size required to support given programs in the case of specialist working full time in a particular field. (A lower value of assurance than 95% is very likely acceptable to administrators and parents). Other relationships are also readily available. For example if the chance of a student taking French was 5 times as great, e.g., p = .05 the student population required would be 1/5 as large, e.g., N = 116/.05 = 2,320.

The other problem to which this chart may be applied is the question of utilization of a program by a given population of students which may then be translated into staff and space requirements. For example, continuing the French illustration, given a student population of N = 10,000 with a chance of p = .01 that any student will take French, then there is 95% assurance that the program will exceed 85 students (reading in Figure 3 on the curve marked 95% the value 85 on the abscissa corresponding to the value Np = 100 on the ordinate). In terms of resource utilization then, assuming that the 100 students represents full utilization, the program will be used more than 85% of capacity with high probability. One could then say, the size of student body would justify the specialization of a French program and in terms of staff requirements, one full-time French teacher.

In general terms assume N and p are known and a P level of assurance has been assigned, then determine an M value as above. The number of teachers, T_s', required will be determined by the following formula,

$$T_s' = \frac{f' M}{W C}$$

where

T_s' = number of teachers required

f' = curriculum requirement of the scheduling of the course of instruction in periods per week

M = number of students that will be exceeded with probability P

W = workload per teacher in periods per week

C = class size in number of students

Only integral values of T_s' will be considered. The particular method of rounding may be specified.

The maximum number of students that choose a program will be used in the determination of space requirements. For example, in the above illustration we have determined that there is a 95% chance that the program will exceed 85 students. For the determination of space requirements the statement that there is only a small chance that the student size will exceed a given number of students is of interest. Referring again to Figure 3 on the curve marked 5%, corresponding to a value of Np = 100 on the ordinate a value of 117 is read on the abscissa. This is interpreted as, there is only a 5% chance that this number of students will be exceeded. Space requirements may therefore be based on this value (or a similar value based on some appropriate small probability level such as .01 or .005.

Since we have previously determined the number of teachers T_s' based on the above formula for the particular curriculum element, the new actual class size, C, is, using now the integral value of T_s',

$$C = \frac{f' \cdot M}{W T_s'}$$

and the space requirement is

$$T_A' = T_s' Cq$$

where C is the class size based on the integral value of T_s' and q is the floor area per pupil factor previously described.

These procedures will be followed for each curriculum element or program that may be defined in terms of the above input parameters and where the contributions of the program are considered relevant to the generation of benefits and/or costs. The example worked above as illustrative of the general procedure was conservative in the determination of teacher requirements and liberal in the determination of space requirements. These may be varied through the selection of the assurance level P for any particular application.

Summary

In summary the school model generates the following outputs,

1. Facilities: Total Floor Area, T_A, Required

$$T_A = \frac{A_c + A_e}{1 - r}$$

$$= \frac{1}{1 - r}\left[\sum_{j=1}^{3} N_i\, g_i + \sum_{j=1}^{n} (f_j/p_j)\, N_j\, q_j\right]$$

2. Staffing: Total Staff, T_s, Required

$$T_s = \sum_{u=1}^{3} \sum_{v=1}^{8} e_{v,u}\, N_u^{f_{v,u}}$$

3. Special Staff, T_s', Requirements

Given N, p, P determine M, then

$$T_s' = \frac{f'\, M}{W\, C}$$

4. Special Space, T_A', Requirements

Given N, p, P determine M, then

$$T_A' = T_s'\, C\, q$$

In terms of educational policy input, the following may be examined:

(1) facility requirements in terms of total school plant size and functional space allocation

(2) staffing requirements by number and occupational categories

(3) special program requirements in terms of staff and space

(4) staff and space implications of scheduling modifications

The output of this model will provide substantial input into the cost and effectiveness submodels.